

A PRELIMINARY INVESTIGATION OF THERMAL ICE PRESSURES
IN CANAAN STREET LAKE, NH, 1983

by

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Abstract

Measured ice stress data are needed to verify and improve thermal ice thrust prediction models used in estimating ice forces on dams, bridge piers, locks, and other hydraulic structures. During February and March, 1983, thermal ice pressures were measured in the ice on a small lake in central New Hampshire. Even though the ice sheet was relatively warm and only exhibited small changes in temperature, stresses up to 200 to 300 kPa were recorded with a newly designed biaxial ice stress sensor. Ice stresses normal and parallel to the shore of the lake were similar.

Given the rate of change of temperature of the ice, ice pressures were calculated for the measurement period using a uniaxial rheological model consisting of a spring and nonlinear dashpot connected in series. Calculated and measured stresses were in good agreement.

Introduction

An understanding of thermal ice pressures in lakes, reservoirs, and rivers is required to effectively design dams, bridge piers, and other hydraulic structures which are subjected to ice thrust. In northern regions thermal ice thrust may even govern the design of concrete gravity dams (Drouin and Michel, 1971). While considerable progress and experience has been acquired during the past fifty years on estimating thermal ice pressures, we still lack confidence in our estimates due to the uncertainty in the rheological behaviour of the ice and lack of reliable field measurements of thermal ice pressures. Field measurements are needed to test and possibly improve existing thermal ice thrust prediction models (Kjeldgaard and Carstens, 1980).

During the past several years, a biaxial ice stress sensor has been under development at Oceanographic Services, Inc., CRREL, and IRAD Gage, Inc. (Johnson and Cox, 1980; Johnson and Cox, 1982; Cox and Johnson, 1983). After the biaxial sensor successfully completed rigorous laboratory tests at CRREL, several sensors were installed in the ice in a nearby lake. The objectives of the field program were to obtain field experience in deploying the sensor as well as some preliminary measurements of thermal ice pressures. This paper presents the results of the field measurement program and calculations of thermal ice pressures, given the rate of change of ice temperature, or ice strain-rate.

Biaxial Ice Stress Sensor

The biaxial ice stress sensor consists of a stiff cylinder made of steel (Fig. 1). It is 20.3 cm long, 5.7 cm in diameter and it has a wall thickness of 1.6 cm (Fig. 2). The ends of the sensor are threaded such

that a rounded end cap can be attached to the lower end of the sensor. Extension rods can also be screwed to the top of the sensor to position the sensing portion of the gauge at any desired depth in the ice sheet. Principal ice stresses normal to the axis of the gauge are determined by measuring the radial deformation of the cylinder wall in three directions. This is accomplished by using vibrating wire technology advanced by IRAD Gage (Hawkes and Bailey, 1973). Three tensioned wires are set 120° from each other across the cylinder diameter. The diametral deformation of the gauge in these three directions is determined by plucking each wire with a magnet/coil assembly and measuring the resonant frequency of the vibrating wires. A thermistor is also placed inside the cylinder to measure the gauge or ice temperature. Both ends of the sensor are sealed to protect the wires and electronics from moisture. The sensor and data logging equipment are fabricated by IRAD Gage in Lebanon, NH.

The magnitude and direction of the principal stresses (p , q , and θ_1) are calculated from the radial deformations (V_{r1} , V_{r2} and V_{r3}) of the sensor by solving three simultaneous equations:

$$V_{r1} = A(p+q) + B(p-q)\cos 2\theta_1$$

$$V_{r2} = A(p+q) + B(p-q)\cos 2(\theta_1+60^\circ)$$

and

$$V_{r3} = A(p+q) + B(p-q)\cos 2(\theta_1+120^\circ)$$

where A and B are constants which depend on the gauge geometry and the mechanical properties of the ice and gauge. The sensor is designed such that A and B are relatively insensitive to variations in the ice modulus. Detailed equations are given in Cox and Johnson (1983).

Controlled laboratory tests by Cox and Johnson (1983) demonstrate that the biaxial ice stress sensor has a low temperature sensitivity (5 kPa/°C), a resolution of 20 kPa, and an accuracy of better than 10% under a variety of loading conditions. The sensor is calibrated in a simple hydraulic pressure cell, not in ice.

Field Measurement Program

The field measurement program was performed on Canaan Street Lake, Canaan, NH, during February and March, 1983. On February 14, three biaxial ice stress sensors were installed in the ice, about 25 m from shore on the northeast side of the lake (Fig. 3). The sensors were placed parallel to the shoreline, about 6 m apart. A 6.4 cm diameter hole was drilled through the ice to accommodate each sensor, and the small water annulus between the ice and gauge was allowed to freeze back. During the freezing-in period, each sensor was supported by a special jig. The sensing portion of the gauges was located 0.13 m below the ice surface. At the time of the installation and during the entire measurement program the ice was about 0.36 m thick. After the sensors were installed in the ice, they were covered with snow and a small insulated box.

On February 17 the gauges were connected to a data logger which was located in a house adjacent to the lake. Ice stress and gauge temperature readings were obtained at 30 minute intervals until March 4, when air temperatures remained consistently above 0°C.

During installation of the sensors, ice rafting and buckling were observed to have occurred along the shore at the study site (Fig. ⁴7). A profile of the beach at the site is given in Figure 5.

Field Program Results

The periods of the wires in each of the sensors and the gauge temperatures were recorded at 30 minute intervals during the measurement program. The data were processed using the calibration procedures and equations given in Cox and Johnson (1983) to obtain the magnitude and direction of the principal stresses in the ice. These data were then used to calculate the magnitude of the stress components normal and parallel to the shore. The results from two sensors are given in Figure 6. Figure 7 presents the ice temperature at the depth of the sensing portion of the sensors (0.13 m). From "no-load" measurements taken before and after the field program, the data from one sensor were rejected as the sensor exhibited considerable drift.

Due to unseasonably warm air and ice temperatures, it took about 4 days to freeze-in the sensors. By the end of February 18, it appeared that the stresses associated with freezing-in of the sensors had relaxed to ambient stress levels.

In general, the stress measurements from the two sensors are in close agreement. This is particularly true for the stress components normal to the shore. Parallel to the shore the difference between the stress measurements is greater. Considering the proximity of the sensors to the beach, this may be due to the variation in the shear stress along the shore boundary. Spatial differences in stress may also be due to active cracks in the ice. As the ice sheet was covered with snow, the crack distribution pattern was not studied.

Maximum compressive stresses normal to the shoreline were about 140 kPa, while maximum tensile stresses approached 240 kPa. Parallel to the

shoreline, peak stresses were greater, about 210 kPa in compression and 310 kPa in tension.

Calculated Thermal Ice Pressures - Ice Rheology

The magnitude of thermal ice pressures in an ice cover depend on a number of factors. These include: the rate of change of temperature in the ice; the ice coefficient of thermal expansion; the rheology of the ice; the size and distribution of wet and dry cracks in the ice cover; the ice thickness; and the degree of confinement of the ice sheet (Bergdahl, 1978). The rate of change of temperature of the ice cover depends on the wind speed, air temperature, solar radiation, and depth, density, and albedo of the snow cover. As far as predicting thermal ice pressures is concerned, the ice rheology and the effect of cracks in the ice cover remain the most significant uncertainties (Kjeldgaard and Carstens, 1980).

Several rheological models have been used to calculate thermal ice pressures, given the rate of change of temperature in the ice (Lindgren, 1970; Drouin and Michel, 1971; and Bergdahl, 1978). Lindgren utilized a linear visco-elastic model composed of a Maxwell unit and a Kelvin-Voigt unit connected in series. Drouin and Michel used a nonlinear model considering the number and multiplication rate of dislocations in the ice. Bergdahl applied an ice model consisting of a linear spring and nonlinear dashpot connected in series. Of the three models, Bergdahl's model appears to best describe the behaviour of ice with the least number of unknown parameters (Bergdahl, 1977).

Bergdahl's rheological model may be expressed as

$$\dot{\epsilon} = \frac{\dot{\sigma}}{E} + KD \left(\frac{\sigma}{\sigma_*} \right)^n \quad (1)$$

where ϵ is the ice strain, σ is the ice stress, σ_* is a unit stress (1 Pa), E is the effective elastic modulus of the ice, D is a diffusion coefficient, and K and n are functions of strain-rate and temperature. The effective modulus, rather than Young's modulus, is used in the model, as the effective modulus also accounts for the effects of delayed elasticity which is not considered in the viscous part of the model. In estimating maximum ice thermal pressures in five Swedish Lakes, Bergdahl and Wernersson (1978) chose

$$E = (1 - C\theta) 6.1 \text{ GPa}, \quad (2)$$

where $C = 0.012 \text{ } ^\circ\text{C}^{-1}$ and θ is the ice temperature;

$$D = D_0 \exp (-Q/RT), \quad (3)$$

where

$$D_0 = 9.13 \times 10^{-4} \text{ m}^2/\text{s} ,$$

$$Q = 59.8 \text{ kJ/mole (activation energy for self-diffusion),}$$

$$R = 8.31 \text{ J/mole } ^\circ\text{K (gas constant),}$$

$$T = \text{absolute temperature;}$$

and

$$K = 4.40 \times 10^{-16} \text{ m}^{-2}$$

$$n = 3.651 .$$

Estimates of E , K , and n were based on limited test data on SI ice (c-axis vertical columnar ice) loaded normal to the optic axis at -10°C .

In calculating thermal pressures in an ice cover, Bergdahl and Wernersson assumed that the ice had a low tensile strength. In their computation program the ice pressure was allowed to reach its maximum tension value, and then the stress was set to zero as if the ice had

cracked. During subsequent warming, a conservative estimate of ice pressure was therefore obtained.

Bergdahl's rheological model was used to calculate the ice stress in Canaan Street Lake during this study. As the ice was warm and presumably ductile, the model was modified to allow tensile stresses to accumulate in the ice cover during cooling periods without any ice cracking. As Bergdahl, a finite difference scheme was used to calculate the ice pressure. However, Newton's method rather than successive substitution was used to solve the nonlinear equation. From Bergdahl(1978) we have

$$\sigma_2 = \sigma_1 + \bar{E} \left[\Delta\epsilon - (D_1 K \sigma_1^n + D_2 K \sigma_2^n) \frac{\Delta t}{2 \sigma_*^n} \right] \quad (4)$$

and the subscripts denote the value of the parameters at times 1 and 2.

The ice thermal strain during the time step, Δt , is calculated from

$$\Delta\epsilon = \epsilon_2 - \epsilon_1 = \alpha(T_2 - T_1)$$

where α is the linear coefficient of thermal expansion and T is the ice temperature. Applying Newton's method (Carnahan and Wilkes, 1973) to solve for σ_2 we obtain

$$\sigma_2(\text{NEW}) = \sigma_2 + \frac{\sigma_1 - \sigma_2 + \bar{E} \left[\Delta\epsilon - (D_1 K \sigma_1^n + D_2 K \sigma_2^n) \frac{\Delta t}{2 \sigma_*^n} \right]}{1 + n \bar{E} D_2 K \sigma_2^{n-1} \frac{\Delta t}{2 \sigma_*^n}} \quad (5)$$

Initially, we set σ_2 equal to σ_1 and calculate $\sigma_2(\text{NEW})$. Next, σ_2 is set equal to $\sigma_2(\text{NEW})$ and $\sigma_2(\text{NEW})$ is recalculated. Several interactions are performed until $\sigma_2(\text{NEW})$ is approximately found to be equal to σ_2 . The stress at the next time step is determined by setting σ_1 to σ_2 and repeating the above procedure.

The computed stress-time history, using Bergdahl's model and his recommended values for E , K , D , and n , is compared to the measured stresses (Gauge 514) in Figure 8. In general, Bergdahl's model over-predicts compressive and tensile stresses in the ice. While a constrained uniaxial spring and nonlinear dashpot rheological model may not be the best model to describe the thermal expansion of an ice sheet, the difference between the measured and calculated stresses can be explained partially by Bergdahl's selection of E , K , and D . Bergdahl chose an effective ice modulus of about 6 GPa to describe the elastic behaviour of the ice. For the low strain-rates typical of thermal expansion, the effective modulus of ice is expected to be much lower (Weeks and Mellor, 1983). Work by Traetteberg et al. (1975) on the modulus of S2 ice suggest that a modulus of 4 GPa may be more appropriate. Bergdahl also uses the Arrhenius equation to describe the effect of temperature on the creep rate of ice (Eq. 3). While this assumption appears to be justified for the creep of polycrystalline ice at temperatures below -10°C , it does not appear to be warranted at higher temperatures. Above -10°C , a plot of $\ln \dot{\epsilon}$ vs $1/T$ is strongly nonlinear, indicating that the creep activation energy, Q , is not constant but increases with increasing temperature (Mellor, 1980). At higher temperatures, deformation processes in addition to diffusion mechanisms come into play. These include recrystallization, grain boundary melting, and grain boundary shear. The product KD in Eq 1 needs to be re-evaluated for ice thermal expansion problems.

Bergdahl used data on the mechanical properties of S1 and at -10°C given in Drouin and Michel (1971) to determine the values of K , D and n . Drouin and Michel also present data on the mechanical properties of S1 ice

at -30 and 0°C. These results were used to define a new function A(T) which describes the temperature dependence of the creep rate of Sl ice, such that,

$$\dot{\epsilon} = \frac{\dot{\sigma}}{E} + A(T) \left(\frac{\sigma}{\sigma_*} \right)^n \quad (6)$$

where

$$A(T) = \frac{\dot{\epsilon}}{(\sigma_{\max}/\sigma_*)^n}$$

in a constant strain-rate test, or

$$A(T) = \frac{\dot{\epsilon}_{\min}}{(\sigma/\sigma_*)^n}$$

in a constant load test. Both types of tests can be used to evaluate A(T) for ice, considering the correspondence between these tests described by Mellor and Cole (1982).

From the curves presented by Drouin and Michel, values of σ_{\max} were obtained for a strain-rate of $2 \times 10^{-8} \text{ s}^{-1}$. The interpolated values from their curves are given in Table 1 and $\ln A(T)$ is plotted against $\ln T$ in Figure 9. To provide a linear fit of the data, it was assumed that the 0°C tests were performed at -1°C. This may not be an unreasonable assumption as it is exceedingly difficult to perform well controlled tests at 0°C. By adjusting the data

$$A(T) = B \left(\frac{T}{T_*} \right)^m$$

where T_* is a unit temperature, $B = 2.46 \times 10^{-29} \text{ s}^{-1}$, and $m = 1.92$.

After suitable values of E and A(T) were determined, Equation 6 was then used to calculate the thermal stresses in the ice during the measure-

ment program. The results are presented in Figure 8. By using more appropriate values of E and $A(T)$ better agreement is obtained between the calculated and measured results. In fact, it is surprising that the agreement is as good as it is, considering the paucity of data on the mechanical properties of columnar ice above -10°C .

Conclusions

While reasonable agreement was obtained between measured and calculated stresses using a modified form of Bergdahl's model, considerably more work needs to be done before we can predict thermal ice pressures with confidence. Additional tests on the mechanical properties of columnar ice at high temperatures and low strain-rates need to be performed to define a suitable rheological model for ice pressure calculations. Criteria must be developed to predict cracking and buckling of the ice cover. The effect of biaxial restraint needs to be further evaluated. Finally, measured stress and strain data must be obtained for larger variations in ice thickness and temperature to test and verify future thermal ice pressure models.

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Table 1. Values of σ_{\max} for horizontal SI ice specimens at different temperatures and corresponding values of $A(T)$ for $n = 3.7$.

CRREP RATE TEMPERATURE DEPENDENCE FACTOR, $A(T)$

| <u>Temperature</u> (°C) | σ_{\max} (MPa) | <u>$A(T)$</u> |
|----------------------------|--------------------------|--------------------------|
| 0 | 0.45 | 2.42×10^{-29} |
| -10°C | 1.46 | 3.11×10^{-31} |
| -30°C | 2.65 | 3.42×10^{-32} |

Figures

Figure 1: Biaxial ice stress sensor.

Figure 2: Schematic of biaxial ice stress sensor.

Figure 3: Map of Canaan Street Lake showing location of ice stress sensors.

Figure 4: Ice rafting and buckling along the shore at the measurement site.

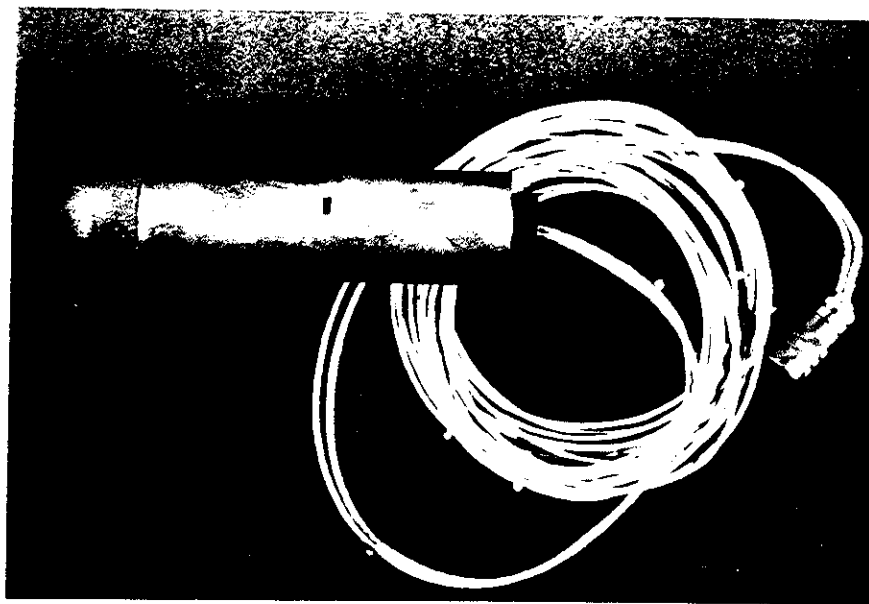
Figure 5: Beach profile at the study site.

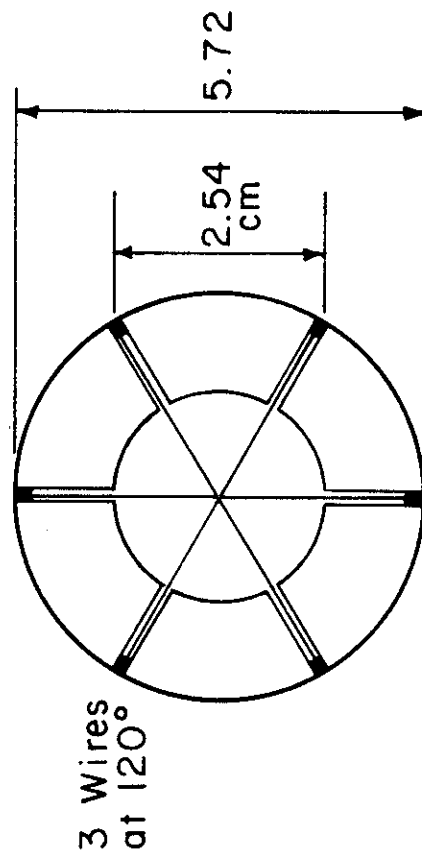
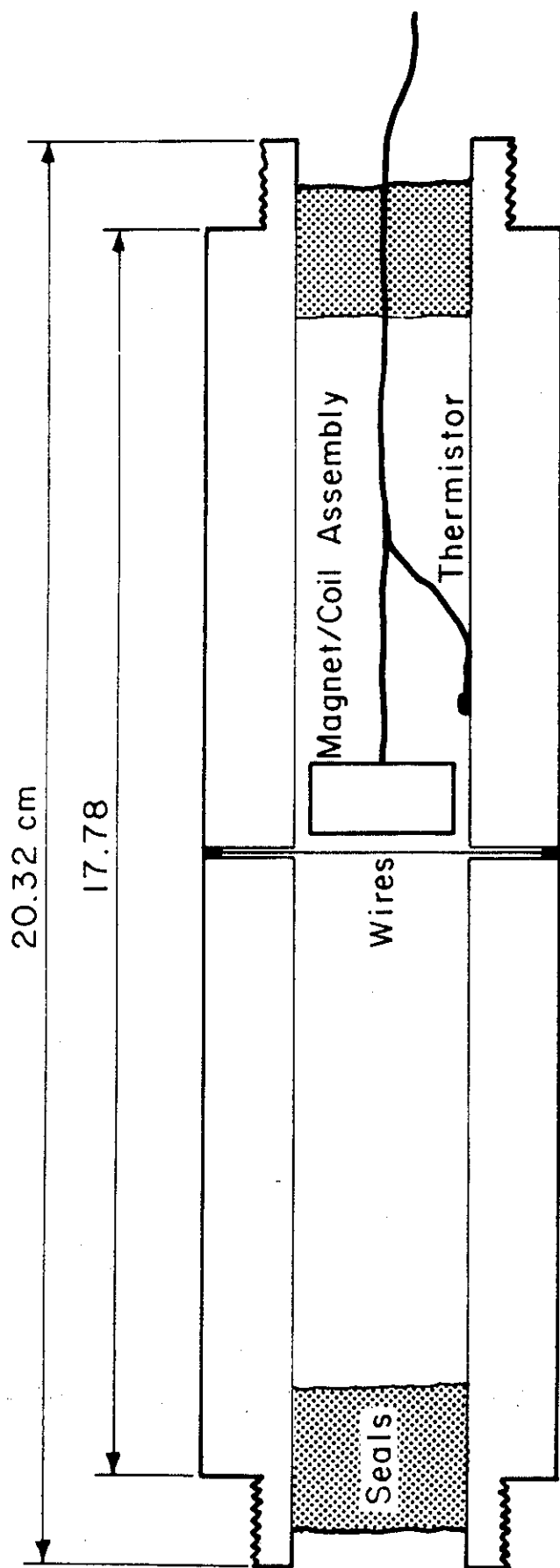
Figure 6: Measured ice stresses in the ice on Canaan Street Lake, NH.

Figure 7: Ice temperature at the depth of the sensing portion of the gauge.

Figure 8: Measured (Gauge 514) versus calculated ice stresses.

Figure 9: Creep rate temperature dependence factor, $A(T)$, versus temperature.





BIAXIAL ICE STRESS SENSOR

